Positron-Cyclotron Maser for the Core Emissions from Pulsars

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ABSTRACT

We use the cyclotron-maser theory to explain the core emission from the magnetosphere of pulsars. As a kind of direct and efficient maser type of emission, it can give rise to escaping radiation with extremely high brightness temperature and narrow angle with respect to the magnetic axis. We find that the growth rates and real frequencies of the O-mode electromagnetic wave propagating parallel to the magnetic fields depend on the ratio of the plasma frequency ω_p and the gyrofrequency ω_b rather than the plasma frequency alone, as described by other models. The emission takes place in the region where the magnitude of ω_p/ω_b is 10^{-2}. The corresponding altitude is about a few decades of neutron star radius, where the magnetic field strength is about $10^6 - 10^8 G$. The qualitative spectrum and the lower frequency cut-off of the radio emission is obtained by this model.

Subject headings: plasmas — radiation mechanisms: non-thermal — pulsars: general — waves

1. Introduction

There is still no satisfactory model for the radio-emission for pulsars despite more than two decades of theoretical effort. The radio-emission mechanisms for pulsars are unclear because the plasma parameters in the pulsar magnetosphere are poorly known. The properties of a plasma in superstrong magnetic field as high as $10^{12}G$ on the surface of the neutron star is not familiar to us as well.

The extremely high brightness temperatures observed from radio pulsars can only be obtained by the coherent motion of the charged particles in magnetized plasmas. The radiation intensity from coherent emission is proportional to N^2 rather than N (the number of particles) as compared with incoherent emission (Melrose 1990). So the incoherent emissions such as cyclotron radiation, synchrotron radiation or inverse-Compton scattering should not be regarded as the favored mechanism for pulsar radio emission. Several candidates of coherent emission mechanisms such as coherent curvature radiation, free electron maser emission, relativistic plasma emission and cyclotron maser emission have been suggested. The coherent curvature emission was the first one put forwards to explain the pulsar radio emission. The most serious difficulty lies mainly in that the bunching instability can not afford any velocity dispersion. The relativistic plasma emission is not a kind of escaping mode itself. A conversion mechanism like nonlinear wave-wave interaction is necessary to couple the plasma turbulence into escaping radiation, but the conversion process limits the radiation efficiency. The free electron maser seems attractive primarily because it is a direct maser emission. But the weakness is that a large amplitude electrostatic wave should be excited, which acts as wiggle field in the free electron laser. The generation of the required parallel electric fields is not clear and the efficiency decreases strongly with increasing Lorentz factor of the energetic particles (Melrose 1986).

Cyclotron instability developping near the light cylinder of the pulsar and other plasma processes, which may be responsible for the generation of radio emission of pulsars, were

investigated by the Geogian group (Machabeli & Usov 1989, 1979; Kazbegi et al. 1991). The source region based on their models seems too further from the star as compared to the recent observation results, i. e. , heights ranging between 1 and 2% of the light-cylinder radius (Hoensbroech & Xilouris 1996).

The cyclotron maser mechanism has been successfully used to interpret planetary radio emissions (e.g. Aurora Kilometer Radiation from the earth), solar decimeter spike bursts, and radio emissions from extragalactic jets (Wu 1984; Yoon & Chang 1989; Ma & Wang 1995). The core emissions of radio pulsars was first studied by Zhu et al. (1994) and Wang et al. (1989). Recently, a number of techniques including pulsar timing, scintillation, pulsar-width narrowing and polarization were developped to derive emission heights (Melrose 1996; Hoensbroech & Xilouris 1996; Kijak & Gil 1996; Kramer 1995). The emission height of a few decades of a neutron star radius is derived for many pulsars. Ginzburg et al. (1975) first pointed out that the high intensity of pulsar radiation might be due to an amplification by maser outside the source rather than in the source. But no such an object has been found by now. According to the cyclotron-maser theory, the electromagnetic wave is amplified inside the source, and it is a kind of escaping wave itself, i.e., masing takes place inside the source.

In this paper, we use cyclotron maser theory in the magnetosphere of the pulsar and calculate the growth rates of micro-instabilities at different altitudes in the magnetosphere of neutron stars. We find that the growth rates of the instability depends on the ratio between the plasma frequency ω_p and the gyrofrequency ω_b rather than the plasma frequency itself. The cyclotron maser emission arises only in the region where the value of ω_p/ω_b falls between 0.01 and 0.1 if the Lorentz factor of the superthermal particles is $10^3 - 10^4$. On the other hand, the ratio ω_p/ω_b depends on the altitude above the neutron star. So we can reduce the emission altitudes according to the regions where the growth rates are positive. The emission altitudes obtained by this model are consistent with the

Radius-to-Frequency Mapping (RFM) observed recently by the Effelsberg 100m radio telescope (Kramer et al. 1996; Kijak et al. 1996; Hoensbroech et al. 1996). In addition, we can get the low-frequency cut-off from this model.

2. The parameters in the magnetosphere of neutron stars

The magnitude of the magnetic field on the surface of the neutron star can be estimated from the observed period derivative as following (Manchester & Taylor 1977)

$$B_0 = 3.2 \times 10^{19} (P\dot{P})^{1/2} \sim 10^{12} G, \tag{1}$$

where P is the rotation period of the neutron star in seconds, and $\dot{P}$ is the time derivative of P. According to the polar cap model of pulsars, a very strong electric field associated with enormous potential drop ($V \simeq 10^{12} - 10^{16} V$) is available just above the surface of the neutron star owing to the rotating magnetic field. So a double layer appears near the surface of the neutron star. The charged particles can be accelerated up to very high energy with Lorentz factor as large as $\gamma_0 \simeq 10^7 - 10^8$ in the double layer. When those primary particles move along the magnetic lines, they emit gamma photons and lose their energy until Lorentz factor falls to about $\gamma_p = 10^3 - 10^5$ (Beskin et al. 1993). At that moment, the primary particles reach the altitude in the magnetosphere of few decades of neutron star radius where the cyclotron maser growth peaks. It is these superthermal primary particles which excite the cyclotron maser instability in the magnetosphere of the pulsar. The energy of the gamma photons emitted by the energetic primary particles is high enough to decay into pairs as they propagate across the magnetic field lines. We call those secondary particles the stream plasma.

According to the Sturrock-Ruderman-Sutherland model, the stream plasma frequency is

(Manchester & Taylor 1977, Wu & Chian 1995)

$$\frac{\omega_p(r)}{\gamma_s^{1/2}} = \left(\frac{8e\gamma_p\Omega B_0}{m_e c}\right)^{1/2}\left(\frac{R}{r}\right)^{3/2}, \tag{2}$$

where R is the neutron star radius, γ_s the Lorentz factor of the stream plasmas, and Ω is the angular rotation frequence of the pulsar. We adopt $\gamma_s = 30 - 100$ in our calculation (Beskin et al. 1993). For a dipolar magnetic field, the gyrofrequency of the stream electrons or positrons is

$$\omega_b(r) = \frac{eB_0}{\gamma_s m_e c}\left(\frac{R}{r}\right)^3. \tag{3}$$

Eq. (2) and (3) are different from the expressions in the book of Manchester & Taylor by the factor of γ_s, because we consider that the background plasma produced by cascade multiplication outside the double layer is relativistic stream too. This improvement was also made by other authors (Wu & Chian 1995, Beskin et al. 1993). The Lorentz factor of the stream plasma is of order about 100.

We can derive the ratio of plasma frequency and cyclotron frequency

$$\frac{\omega_p(r)}{\omega_b(r)} = 1.7 \times 10^{-9}(\gamma_p\gamma_s^3)^{1/2}(PB_{012})^{-1/2}r^{3/2}, \tag{4}$$

where the magnetic field B_{012} in unit of $10^{12}G$ and r in unit of the neutron star radius R. In Sec. 4, We will illustrate that the growth rates strongly depend on the parameter $\omega_p(r)/\omega_b(r)$, which is a function of the radius from the center of the neutron star. The cyclotron maser can only take place in the region where the magnitude of the ratio $\omega_p(r)/\omega_b(r)$ is about 0.01-0.10. Once the ω_p/ω_b is determined from the numerical calculation, the emission altitudes can be deduced from Eq. (4). We find that the emission altitudes range in a few decades of neutron star radius where the magnetic field is about $10^6 - 10^8G$.

3. Positron-cyclotron maser emission

We consider two components of the plasma in the magnetosphere of pulsars: (1) the high energy primary paricles, $\gamma_p = 10^3 - 10^5$; (2) the cold stream plasmas, $\gamma_s = 30 - 100$. As a beam of energetic positrons streams through the plasma layer above the polar cap of a pulsar, the electromagnetic waves can be directly amplified due to wave-particle resonant interaction. It is illustrated that only the O-mode wave propagating parallel the ambient magnetic field is a possible candidate for the core emission (Zhu et al. 1994). The dispersion relation of the electromagnetic wave propagating along the ambient magnetic field is

$$\begin{aligned} N^2 &= 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega} \int d^3p \frac{1}{\gamma_\alpha \omega \pm \omega_b - k_\parallel p_\parallel} \frac{p_\perp^2}{2\gamma_\alpha} \times \\ & \left(\frac{\gamma_\alpha \omega - k_\parallel p_\parallel}{p_\perp} \frac{\partial}{\partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel} \right) F_\alpha(p_\perp, p_\parallel), \end{aligned} \tag{5}$$

where α denotes different components in the plasmas, and $F_\alpha(p_\perp, p_\parallel)$ is the distribution function. To sum over α types of components, i. e. primary particles and magnetosphere plasma streams, we get

$$\begin{aligned} N^2 &= 1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_b)} + \pi \frac{\omega_{pp}^2}{\omega} \int_{-\infty}^{\infty} dp_\parallel \int_0^\infty dp_\perp \frac{1}{\gamma_p \omega \pm \omega_b - k_\parallel p_\parallel} \\ & \frac{p_\perp^2}{\gamma_p} \left(\frac{\gamma_p \omega - k_\parallel p_\parallel}{p_\perp} \frac{\partial}{\partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel} \right) F_p(p_\perp, p_\parallel), \end{aligned} \tag{6}$$

here ω_{pp} is the plasma frequency of the primary particles. According to the wave-particle resonant condition

$$\gamma_p \omega_r - \omega_b - k_\parallel p_\parallel = 0, \tag{7}$$

We can see that only the positrons with positive ω_b have a contribution to the cyclotron maser emission. We write the frequency of the excited wave as $\omega = \omega_r + i\omega_i$, here ω_i devotes the growth rate of the wave. The real part ω_r meets the dispersion equation of the background plasma. The dispersion relation of the O-mode wave propagating parallel to

the ambient magnetic field B_0 with frequency

$$N^2 = \frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega_r(\omega_r + \omega_b)} \tag{8}$$

By use of the resonant condition and the dispersion relation, this yields

$$\begin{aligned} \omega_i &= \frac{\pi^2}{G}\frac{\omega_{pp}^2}{\omega_r}\int_{-\infty}^{\infty} dp_\parallel \int_0^\infty dp_\perp \delta(\gamma_p\omega - \omega_b - k_\parallel p_\parallel)\frac{p_\perp^3}{\gamma} \times \\ & \left(\frac{\gamma_p\omega - k_\parallel p_\parallel}{p_\perp}\frac{\partial}{\partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel}\right) F_p(p_\perp, p_\parallel), \end{aligned} \tag{9}$$

where

$$G = 2 - \frac{\omega_p^2\omega_b}{\omega_r(\omega_r + \omega_b)^2}$$

we consider the primary positrons to be a hollow-beam distribution

$$\begin{aligned} F_p(p_\perp, p_\parallel) &= \frac{exp(-\xi_0^2)}{1 + \xi_0 Z(\xi_0)}(\pi\alpha_\perp^2)^{-1}(\pi\alpha_\parallel^2)^{-1/2} \\ & exp\left[-\frac{(p_\perp - p_{0\perp})^2}{\alpha_\perp^2} - \frac{(p_\parallel - p_{0\parallel})^2}{\alpha_\parallel^2}\right] \end{aligned} \tag{10}$$

where $\xi_0 = -ip_{0\perp}/\alpha_\perp$, $\alpha_\perp$ and $\alpha_\parallel$ denote the characteristic velocity spreads, and the average momenta of primary particles are

$$\frac{p_{0\perp}}{mc} \simeq \sqrt{\gamma_p\,(2 + \gamma_p)}\sin\psi, \tag{11}$$

$$\frac{p_{0\parallel}}{mc} \simeq \sqrt{\gamma_p\,(2 + \gamma_p)}\cos\psi, \tag{12}$$

where ψ is the pitch angle of the particles. It must be pointed out that the energetic particles lose their momentum component perpendicular to the magnetic field because of synchrotron losses. But the energy of the cyclotron maser comes from the perpendicular momentum of the primary positrons. As a result, the energetic particles acquire non-zero pitch angle. But the quasi-linear wave-particle interaction brings about the diffusion of particles in the momentum space both along and across the magnetic field. The waves,

which are not escaping mode in the plasmas, may be excited by cyclotron instability, beam instability et al. (Lominadze et al. 1983). On the other hand, considering the initial energy of the primary particles $\gamma_0 \sim 10^7 - 10^8$, their average pitch angle $\psi = 5°$ and the magnetic field on the surface of the neutron star $B_0 = 10^{12}G$, traversing one scale height of the magnetosphere $\sim r$, we get $r/R > 100$ when the final energy is $\gamma \sim 10^4$. Hence, in order for the model to be plausible, the wave particle interaction should be effectice over a distance smaller than a scale height and occur exactly where the maser growth rate peaks. We adopt a finite value of pitch angle $\psi = 5°$ in our calculations. Finally the growth rate is written as

$$\frac{n_s}{n_p}\frac{\omega_i}{\omega_b} = -\frac{2\sqrt{\pi}A}{G}\left(\frac{\omega_p}{\omega_b}\right)^2\left(\frac{\omega_r}{\omega_b}\right)^{-2}\int_{p_-}^{p_+} dp_\parallel \bar{p}_\perp \times$$

$$\left[\frac{\bar{p}_\perp - p_{0\perp}}{\alpha_\perp^2} + N\left(\frac{\omega_r}{\omega_b}\right)\frac{\bar{p}_\perp}{C}\frac{p_\perp - p_{0\parallel}}{\alpha_\parallel^2}\right]$$

$$exp\left[\frac{(\bar{p}_\perp - p_{0\perp})^2}{\alpha_\perp^2} - \frac{(p_\parallel - p_{0\parallel})^2}{\alpha_\parallel^2}\right] \qquad (13)$$

where n_s and n_p are the densities of the stream plasma and the primary particles respectively.

$$A = \frac{C^2}{\alpha_\perp^2\alpha_\parallel}\frac{exp(p_{0\perp}^2/\alpha_\perp^2)}{1+\sqrt{\pi}(p_{0\perp}/\alpha_\perp)[1+erf(p_{0\perp}/\alpha_\perp)]exp(p_{0\perp}^2/\alpha_\perp^2)}$$

erf is the error function, and

$$\bar{p}_\perp = C\sqrt{(N^2-1)(p_\parallel^2/C^2) + 2N(\omega_r/\omega_b)^{-1}(p_\parallel/C) + (\omega_r/\omega_b)^{-2} - 1}$$

The lower and upper limits of the integral are

$$p_\pm = C\left[\frac{N}{1-N^2}\left(\frac{\omega_r}{\omega_b}\right)^{-1} \pm \frac{1}{1-N^2}\sqrt{N^2 - 1 + (\omega_r/\omega_b)^{-2}}\right]$$

In the next section, we will use Eq. (13) to calculate the growth rates numerically and

derive the radiation altitudes.

4. Numerical results

We set the momentum spreads of the primary particles $\alpha_\parallel/mc = \alpha_\perp/mc = 50$, and the pitch angle of the primary particles $\psi = 5°$. The energies of the primary particles equal $\gamma_p = 10^3$ and 10^4 in the calculations respectively. The pitch angle can be even smaller although it will decrease the growth rates of the amplified waves. The results are displayed in Fig. 1 and Fig. 2.

In Fig.1, we show the growth rates of the cyclotron maser instability for different values of ω_p/ω_b as $\gamma_p = 10^3$. When the ratios of ω_p/ω_b. increases from 0.03 to 0.044, the growth rates increase while the real frequencies decrease continuously. If the ratios of ω_p/ω_b increase further, the growth rates decrease but the real frequencies increase. The growth rates reach a maxium value at $\omega_p/\omega_b = 0.044$, which corresponds the peak of the radiation spectrum. So The spectral peak lies at about $\sim 0.01\omega_b$ according to Fig.1 and Fig. 2., a lower frequency cut-off appears beyond the turn-over point. It is of interesting that the frequency decreases at first, then increases beyond some point as the altitude rises. In fact, in contrast to most pulsars, PSR B0525+21 and PSR B0740-28 emit the higher frequencies in higher emission altitudes (Hoensbroech & Xilouris 1997).

EDITOR: PLACE FIGURE 1 HERE.

In Fig.2, all the parameters are the same as in Fig.1 except for $\gamma_p = 10^4$. The significant emissions arise in the region where the ratios of ω_p/ω_b fall in [0.010, 0.013]. We can see that the frequency decreases monotonously with radius in this case. When $\omega_p/\omega_b = 0.013$, the growth rate gets its maxium. If we adopt $\omega_p/\omega_b = 0.014$, the growth rate is almost

zero at any real frequency, i. e. , the frequency spectrum is cut-off suddenly. Because $\omega_p/\omega_b \propto r^{3/2}$, we can see from Fig. 2 that the lower frequencies arise from higher altitudes in the magnetosphere of neutron star in case when the Lorentz factor of the energetic particles is 10^4. herefore, we can conclude that the Lorentz factors of primary particles in the emission regions are larger than 10^3 for most pulsars.

EDITOR: PLACE FIGURE 2 HERE.

It should be mentioned that all the frequencies are normalized by local gyrofrequency which is a function of the radius r. From Eq. (4) we can derive the absolute emission altitudes according to the ratios ω_p/ω_b with positive growth rates. For a comparison to our results, we analyzed three samples, i. e. , PSR 0833+45, PSR 1133+16, and PSR 1929+10. In Table 1, we reduce the emission altitudes of the three pulsars in case of $\gamma_p = 10^3$ and 10^4 respectively.

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It is inspiring to compare our results with the observed results by Radius-to-Frequency Mapping (Kramer 1995, Melrose 1996). In Table 2 we display the emission altitudes r in unit of the neutron star radius R, frequencies f in GHz, and growth rates in $n_p/n_s s^{-1}$ for three pulsars when $\gamma_p = 10^3$. We can see that the emission is created within a very compact region. The radius to frequency mapping obtained by this model is much steeper than observed. For example, from Table 2, we find that changing the radius by a factor of $\sim 10\%$ leads to a change in frequency by a factor of ~ 40. It can probably be improved when the nonlinear saturate processes of the instability are considered. The altitude is about a few decades of neutron star radius which is qualitatively coincident with the oberved results. If $\gamma_p = 10^4$, the emission should be closer to the surface of the neutron star, and the emission

frequency decreases monotonously as the height increases. Therefore, we can predict that the Lorentz factor of primary particles should be larger than 10^3 if the observed frequency fellows the power law $\omega_r \sim r^\nu$ (Hoensbroech & Xilouris 1996, Kijak & Gil 1996, Kramer 1995).

EDITOR: PLACE TABLE 2 HERE.

5. Conclusion remarks

The emission height is related to the ratio of local plasma frequency and local gyrofrequency, not to the local plasma frequency alone as described by other models (Beskin et al. 1993, Melrose 1992). The Radius-to-Frequency Mapping property exists because the ratio of local plasma frequency and local gyrofrequency depends on the altitude above the neutron star. Significant amplification of escaping radiation takes place in the area where the ratio ω_p/ω_b is of magnitude about 10^{-2} and the relevant altitude is a few hundreds kilometers. Our results are consistent with the Radius-to-Frequency Mapping model and we can compare them with the observed results (Hoensbroech et al. 1996, Kijak et al. 1996, Kramer 1995). It should be pointed out that the polarization of the O-mode waves in our model is circular polarization. However, if we consider the e^--e^+ plasmas in the magnetosphere, the polarization of normal waves would be linear polarization (Beskin, Gurevich, & Istomin 1993). According to the relation between the growth rate and the real frequency (see Fig. 2), we can analyze the emitting spectrum qualitatively with this linear model. A lower frequency cut-off is obtained in this model. To get a quantitative spectral shape, however, we must consider that the nonlinear wave-wave interactions in plasma limits the growth of the instabilities. We can only predict the spectral index after considering the nonlinear saturation processes.

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Table 1: Emission altitudes R_{em} in unit of neutron star radius R of three pulsars for $\gamma = 10^3$ and 10^4 respectively.

Sources	P	$\dot{P}$	B_0	ω_p/ω_b		R_{em}	
	s^{-1}	$10^{-15}s^{-1}$	$10^{12}G$	$\gamma_p = 10^3$	$\gamma_p = 10^4$	$\gamma_p = 10^3$	$\gamma_p = 10^4$
PSR 0833+45	0.08930	124.7	1.055	1.710^{-4}	$5.5\ 10^{-4}$	41 ± 6	4 - 8
PSR 1133+16	1.023	3.733	6.180	2.110^{-5}	$6.8\ 10^{-5}$	163 ± 15	18 - 33
PSR 1929+10	0.2265	1.157	1.619	$8.8\ 10^{-5}$	$2.8\ 10^{-4}$	63 ± 5	7 - 13

Table 2: Emission altitudes r in R, frequencies f in GHz and the maxium growth rates $\omega_i/\omega_b \times (n_s/n_p)$ of the three pulsars in case of $\gamma_p = 10^3$.

PSR0833+45			PSR1133+16			PSR1929+10		
r	f	ω_{imax}	r	f	ω_{imax}	r	f	ω_{imax}
35	1.2	$1.4\ 10^7$	139	0.11	$1.4\ 10^6$	54	0.30	$3.5\ 10^6$
37	0.54	$4.1\ 10^7$	146	0.035	$4.2\ 10^6$	57	0.17	$1.3\ 10^7$
39	0.22	$3.5\ 10^8$	157	0.02	$3.4\ 10^7$	61	0.07	$1.1\ 10^8$
41	0.03	$4.5\ 10^9$	162	0.003	$3.6\ 10^8$	62	0.01	$1.5\ 10^9$
42	0.07	$3.6\ 10^7$	167	0.007	$3.4\ 10^6$	64	0.02	$1.1\ 10^8$
43	0.35	$1.2\ 10^7$	171	0.034	$1.2\ 10^6$	66	0.11	$3.9\ 10^7$
44	1.0	$3.8\ 10^6$	176	0.095	$3.6\ 10^5$	68	0.30	$1.1\ 10^6$
45	1.2	$1.7\ 10^6$	181	0.13	$1.6\ 10^5$	70	0.37	$5.0\ 10^6$

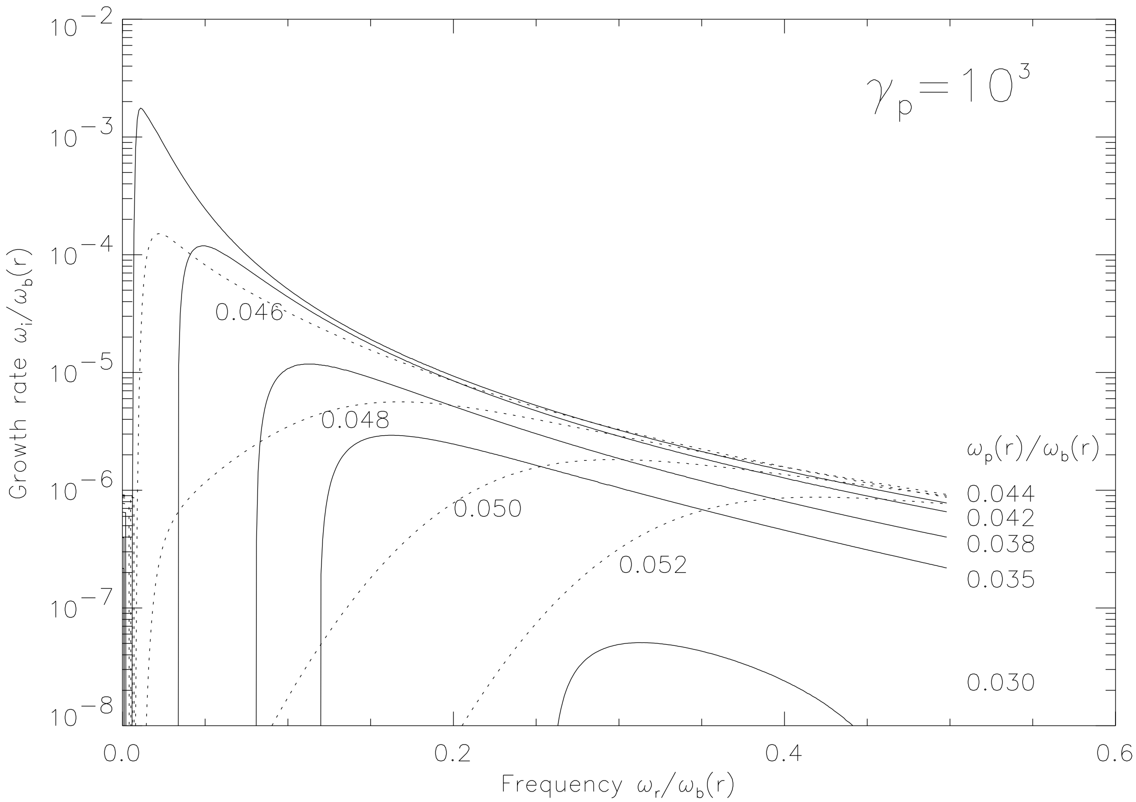

Fig. 1.— The growth rates $(n_p/n_b)(\omega_i/\omega_b)(r)$ *vs.* the frequencies $\omega_r/\omega_b(r)$. for different ratios of plasma frequency and gyrofrequency. All the frequencies are normalized by the local gyrofrequency. The Lorentz factor of the primary particles is 10^3.

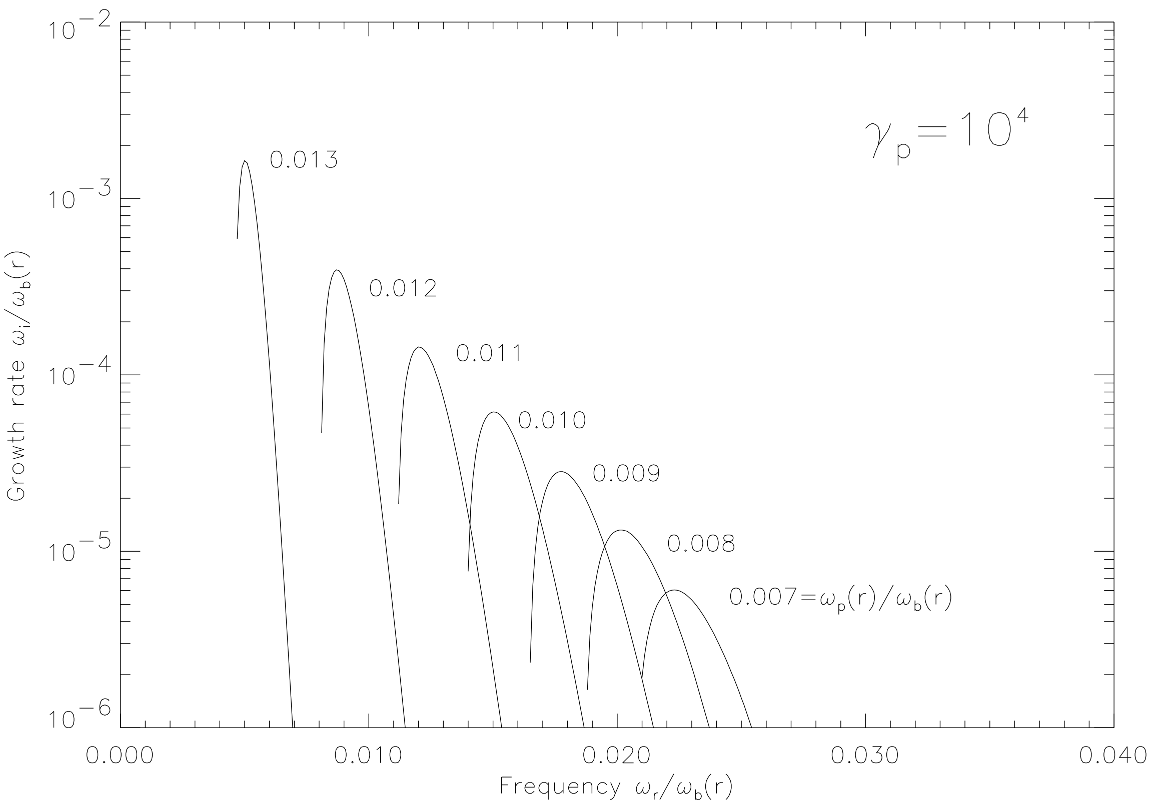

Fig. 2.— The growth rates $(n_p/n_b)(\omega_i/\omega_b)(r)$ *vs.* the frequencies $\omega_r/\omega_b(r)$. for different ratios of plasma frequency and gyrofrequency. The Lorentz factor of the primary particles is 10^4.